VaR
Monte Carlo Simulation

Capital Market Risk Advisors

Monte Carlo Simulation

Monte Carlo is most helpful when some or all assets in a portfolio are not amenable to analytical treatment

1. **Scenario Generation -produce a large number of future price scenarios**
2. **Portfolio valuation - for each scenario, compute a portfolio value**
3. **Summary - report the results of the simulation, either as a portfolio distribution or as a particular risk measure**

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Monte Carlo Simulation
Scenario generation

Monte Carlo begins with the generation of n normal variables with unit variance and correlation matrix Σ.

- Decompose the correlation matrix Σ using the Cholesky factorization, yielding $\Sigma = A^T A$
- Generate an $n \times 1$ vector Z of independent standard normal variables
- Let $Y = AZ$. The elements of Y will each have unit variance with the correlation matrix

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Monte Carlo Simulation Cholesky Decomposition

- Suppose $\Sigma = A^T A$, where A is an upper triangular matrix, how do we find A?

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} a_{11}{}^2 & a_{11}a_{21} & a_{11}a_{31} \\ a_{11}a_{21} & a_{21}^2 + a_{22}^2 & a_{21}a_{31} + a_{32}a_{22} \\ a_{11}a_{31} & a_{21}a_{31} + a_{32}a_{22} & a_{31}^2 + a_{32}^2 + a_{33}^2 \end{bmatrix}$$

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General Result

$$a_{ii} = \left(s_{ii} - \sum_{k=1}^{i-1} a_{ik}^2 \right)^{1/2}$$

$$a_{ij} = \frac{1}{a_{ii}} \left(s_{ij} - \sum_{k=1}^{i-1} a_{ik} a_{jk} \right)^{1/2} \quad j = i+1, i+2, \cdots, N$$

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